

VERIFICATION OF THE MULTI-AXIAL, TEMPERATURE AND TIME DEPENDENT (MATT) FAILURE CRITERION

D. E. Richardson and D. J. Macon

ATK Thiokol, Inc.

Brigham City, UT

e-mail: david.richardson@atk.com, david.macon@atk.com

Introduction

An extensive test and analytical effort has been completed by the Space Shuttle's Reusable Solid Rocket Motor (RSRM) nozzle program to characterize the failure behavior of two epoxy adhesives (TIGA 321 and EA946). As part of this effort, a general failure model, the "Multi-Axial, Temperature, and Time Dependent" or MATT failure criterion^{1,2,3} was developed. In the initial development of this failure criterion, tests were conducted to provide validation of the theory under a wide range of test conditions. The purpose of this paper is to present additional verification of the MATT failure criterion, under new loading conditions for the adhesives TIGA 321 and EA946. In many cases, the loading conditions involve an extrapolation from the conditions under which the material models were originally developed.

Testing was conducted using three loading conditions: multi-axial tension, torsional shear, and non-uniform tension in a bondline condition. Tests were conducted at constant and cyclic loading rates ranging over four orders of magnitude.

Tests were conducted under environmental conditions of primary interest to the RSRM program. The temperature range was not extreme, but the loading ranges were extreme (varying by four orders of magnitude). It should be noted that the testing was conducted at temperatures below the glass transition temperature of the TIGA 321 adhesive. However for the EA946, the testing was conducted at temperatures that bracketed the glass transition temperature.

Theoretical

The Multi-Axial Temperature and Time (MATT) dependent failure model in its most generic form is as follows^{1,2,3}.

$$AP^2J_2 + BPI_1 = 1 \quad (1)$$

J_2 is the second deviatoric stress invariant, and I_1 is the first stress invariant. The shape parameters A and B define the ellipsoidal nature of the failure envelope. These parameters are independent of temperature or time. The

factor P is a multiplier that scales the failure envelope to a proper level for a given temperature and failure time.

This failure criterion is equivalent to the Tsai-Wu⁴ failure model and is equivalent to a modified Drucker-Prager failure model⁵ if a constant P value is used.

For this study, the P factor is found using a linear cumulative damage model approach⁶. One possible expression for the model is:

$$N_\sigma = \left[\int_0^{t_f} \sigma_i^\beta dt \right]^{1/\beta} \quad (2)$$

N_σ and β are experimentally determined failure parameters. σ_i is the time dependent stress during loading. The failure time is t_f . The linear cumulative damage equation is simplified for the following basic loading conditions.

For constant loading rate:

$$t_f = (1 + \beta) \left[\frac{N_\sigma}{\sigma_f} \right]^\beta \quad \sigma_f = N_\sigma \left[\frac{t_f}{1 + \beta} \right]^{-1/\beta} \quad (3)$$

It can be shown that these equations can also be used for a saw-tooth cyclic loading pattern as follows:

$$t_{tot} = (1 + \beta) \left[\frac{N_\sigma}{\sigma_{peak}} \right]^\beta \quad \sigma_{peak} = N_\sigma \left[\frac{t_{tot}}{1 + \beta} \right]^{-1/\beta} \quad (4)$$

where t_{tot} is the total time subjected to cyclic loading and σ_{peak} is the peak stress during the cyclic loading. It is assumed that the minimum load is zero.

Using these relationships and normalizing the A and B parameters, the MATT failure criterion is rewritten to the following form for constant loading rate evaluations:

$$A_\sigma B_\sigma^2 \left(\frac{t_f}{1 + \beta} \right)^{2/\beta} J_2 + B_\sigma \left(\frac{t_f}{1 + \beta} \right)^{1/\beta} I_1 = 1 \quad (5)$$

For cyclic load conditions:

$$A_{\sigma} B_{\sigma}^2 \left(\frac{t_{\text{tot}}}{1+\beta} \right)^{\frac{2}{\beta}} J_2 + B_{\sigma} \left(\frac{t_{\text{tot}}}{1+\beta} \right)^{\frac{1}{\beta}} I_1 = 1 \quad (6)$$

B_{σ} is a combined MATT failure parameter and aids in defining of both shape and size of the failure ellipse. A_{σ} is a slight modification from that given in Equation 1. A_{σ} is a shape parameter that absorbs the linear cumulative damage failure norm seen in Equations 5 and 6.

For TIGA 321 and EA946, the A_{σ} , B_{σ} and the β parameters were previously found to be functions of temperature.²

Experimental

To provide additional model verification, several tests were conducted and compared with MATT predictions. For this study, multi-axial tensile tests were conducted using Tensile Adhesion Button (TAB) specimens. Shear adhesion tests were conducted with the Napkin Ring (NR) test specimen. Non-uniform stress testing was conducted using Tapered Double Cantilever Beam (TDCB) specimens. Sketches of these test geometries can be seen in Figure 1.

In general, tests were conducted under the following conditions (some minor changes were made depending on availability and test capability for some tests). Three test temperatures were used: 21 °C, 27 °C, and 32 °C. TAB and TDCB constant load rate tensile tests were conducted at three rates: 50 cm/min, 1.3 cm/min, and 0.05 cm/min. NR shear adhesion tests were conducted at 200°/min, 2°/min, and 0.02°/min. TAB, TDCB, and NR cyclic loading rates were: 10 Hz, 0.5 Hz, and 0.01 Hz. For the cyclic loading, a sinusoidal loading pattern was used with the minimum load chosen to be 10% of the maximum load. Mathematically, this was modeled with a saw-tooth shape. For all cyclic loading conditions, the maximum load levels were chosen to give reasonable failure times (loads will be seen in the results section). Two to four samples were used for each test condition.

These test conditions differ from those used in the past. Load rates covered four orders of magnitude. Cyclic loading conditions have not previously been evaluated using this failure criterion.

The TDCB test specimen was chosen to give a non-uniform stress distribution in the bondline. Figure 2 shows this linear elastic stress distribution as calculated using finite element analysis.

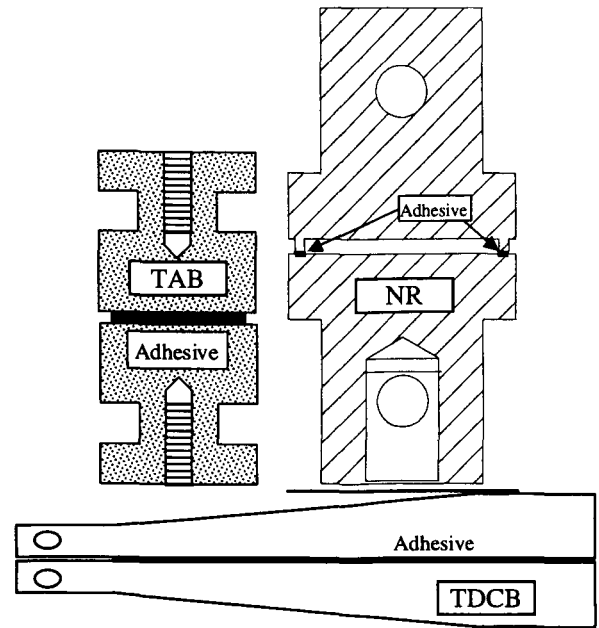


Figure 1: Test Specimens

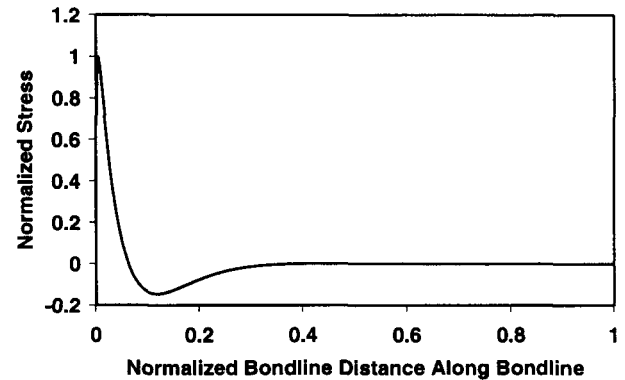


Figure 2: Stress Distribution in TDCB Specimens

Results and Discussion

The testing indicated that, in general the MATT failure criterion accurately predicts failure, even though failure prediction involved extrapolation from the material model database.

TAB Testing

For the constant load rate TAB testing, all rates were faster than that used previously. Correlation was good at the two slower rates (TIGA 321 error: 4%, EA946 error: 24%), but poor for the fastest rate (TIGA error: 53%, EA946 error: 54%). The adhesives appeared to change failure mode at the fast rates (this was evident in the EA946 failure modes). For cyclic loading, the failure model made very good predictions (TIGA 321 error: 8%, EA946 error: 7%). Examples of the data for EA946 can be seen in Figure 3.

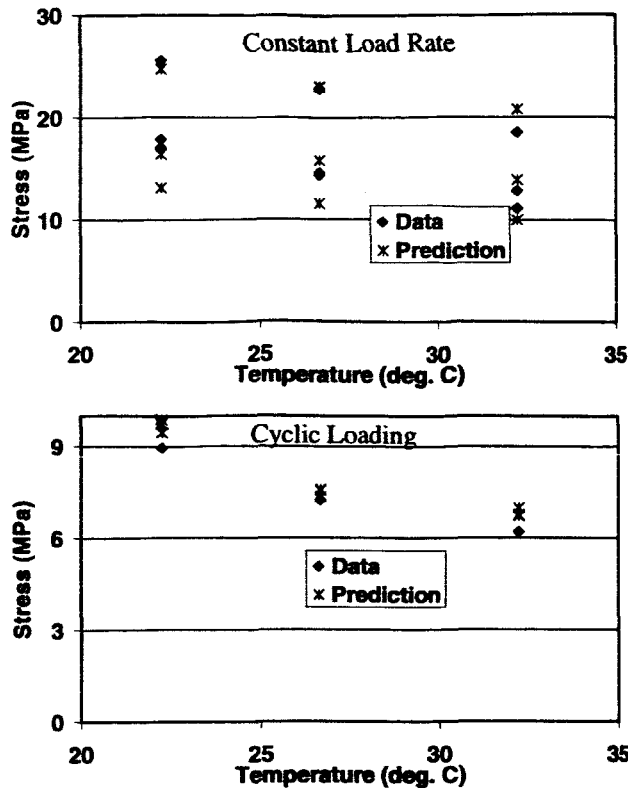


Figure 3: TAB Test Results vs. Predictions for EA946

TDCB Testing

For the constant load rate TDCB testing, the stress distribution was dramatically different than previously tested (see Figure 2). For TIGA 321, the correlation was good at the fastest constant load rate (error: 8%), but poor for the slower two constant load rates and cyclic loading (error: 55%). The predictions were consistently lower than the actual data, most likely caused by plastic deformation (consistent with stress-strain curves), which resulted in a redistribution of stresses. Including plasticity in the stress prediction, leads to very accurate predictions of failure under these conditions. It is felt that the fast rate tests do not have time for plastic deformation or redistribution of stresses.

For EA946, the correlation was poor at the fastest constant load rate (error: 120%), suggesting a shift in failure mechanism at very fast rates. Predictions were fair for the slower two constant load rates and cyclic loading (constant load rate error: 28%, cyclic error: 18%). The data did have indications of stress redistribution as was seen with TIGA 321. Examples of EA946 test data and predictions can be seen in Figure 4.

NR Testing

For EA946, the predictions were fair to good (constant load rate error: 34%, cyclic loading error: 8%). For TIGA, the predictions were fair (cyclic loading error: 33%).

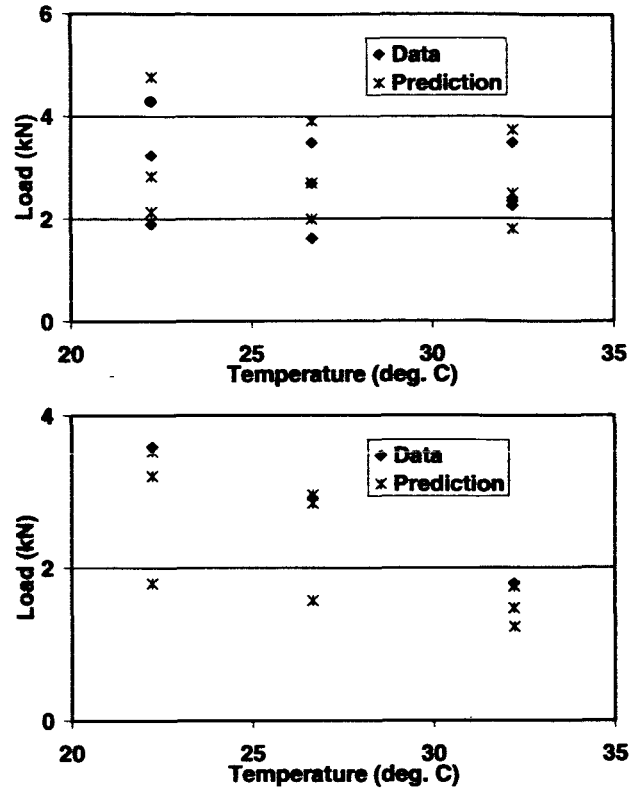


Figure 4: TDCB Test Results vs. Predictions for EA946

Conclusions

Significant testing has been conducted to provide further verification of the MATT failure criterion. Comparison of the test data with predictions has shown that MATT is accurate for a wide range of conditions (including some extrapolation). Not surprisingly at very fast loading rates the failure mechanism changes and significant error is introduced. For non-uniform loading, conservative failure predictions can be made. For accurate predictions, it is necessary to include the effects of plasticity.

References

1. D. E. Richardson, M. L. McLennan, G. L. Anderson, D. J. Macon, and A. Batista-Rodriguez, *J. Adhesion*, 2003, 79, pp. 157-174.
2. D. E. Richardson, G. L. Anderson, and D. J. Macon, *Int J. Adhesion and Adhesives*, 2004, 24, pp 211-217.
3. D. E. Richardson, G. L. Anderson, and D. J. Macon, *AIAA Journal*, (in press).
4. S. W. Tsai and E. M. Wu, *J. Compos. Mater.*, 1971, 5, pp. 58-80.
5. D. C. Drucker and W. Prager, *Q. Appl. Math.*, 1952, 10, pp. 157-165.
6. K. L. Laheru, *J. Propuls. Power*, 1992, 8, pp. 756-759.